14.13 Questions

1. The Fourier transform of a signal $v(t)$ is known to be $V(j\omega) = |\omega|e^{-|\omega|}$ volts/Hz. What will be the approximate amplitude and frequency of the output signal if this signal is applied to an ideal band-pass filter that passes signals of frequencies in the range 0.49 to 0.51 rad/sec?
2. The signal $v(t) = 2\delta(t) + 3e^{-t} + 2e^{-0.3t}\cos 10t + v_S(t)$ is known to have a Fourier transform of $V(j\omega) = 3+2\delta(j\omega)$. What must be $v_S(t)$?
3. The Fourier transform values for a signal $v(t)$ at j2rad/sec and $-j$2rad/sec are $-0.5+j0.7$ and $-0.5-j0.8$ respectively. Explain why $v(t)$ can not be a real function of t?
4. Fourier transform of a rectangular pulse symmetrically located in time-axis is found to cross the frequency-axis for the first time at 100π rad/sec. The dc content in its spectrum is 1 volt/Hz. Find its normalised energy.
5. A time-function $v(t)$ is defined as $v(t) = \sin(1/t)$ in the interval [0,1] and zero elsewhere. Will this function satisfy Dirichlet's conditions?
6. Does unit ramp function satisfy Dirichlet's conditions?
7. The Fourier transform of $v(t)$ has a value of 0.1volt/Hz at $\omega = 0$. $v(t)$ is applied to an initially relaxed inductor of 0.1 H. What is the inductor current magnitude as $t \to \infty$?
8. The value of a real $v(t)$ at $t = 0$ is known to be zero. Explain why its Fourier transform should have a real part that must be negative in some frequency range.
9. The Fourier transform value at $\omega = 0$ for a signal $v(t)$ is $1-j0.7$. Explain why $v(t)$ can not be a real signal?
10. The magnitude spectrum of a signal $v(t)$ is shown in Fig. 14.13-1.Explain why this signal can not be a real function of t ?

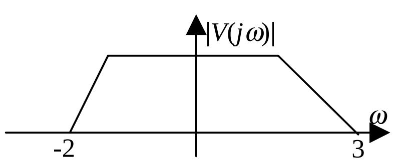

Fig. 14.13-1

11. A signal $v(t)$ is given in Fig. 14.13-2 . Find the Fourier transform of its integral.

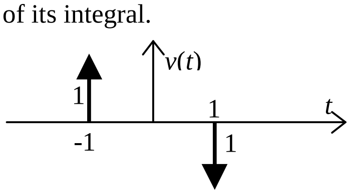

Fig. 14.13-2

12. What is the time-function $v(t)$ if its Fourier transform is $V(j\omega) = 0.5[\cos 2\omega - j\sin 2\omega]$?
13. Find the Fourier transform of $v(t) = 2e^{-3|t|}$.
14. Use the Fourier transform of a rectangular pulse to show that (i) $\int_0^{\infty} \frac{\sin x}{x}\,dx = \pi/2$ (ii) and $\int_0^{\infty} \mathrm{sinc}(x)\,dx = 1/2$.
15. Find the Fourier transform of $te^{-\alpha t}u(t)$ for positive real α starting from the Fourier transform of $e^{-\alpha t}u(t)$.
16. Verify time-domain differentiation property of Fourier transforms for $v(t) = e^{-\alpha t}u(t)$.
17. A signal $v(t)$ has a normalised energy of 0.01 J. What is the normalised energy of $v(3t)$?
18. If $V_o(j\omega) = 10/(j\omega+1)$ in the initially relaxed circuit in Fig. 14.13-3 what is the $v_i(t)$ applied to it?

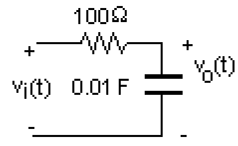

Fig. 14.13-3

19. If $V_o(j\omega) = 10/[(1-\omega^2)+2j\omega]$ in the circuit in Fig. 14.13-3 what is the $v_i(t)$ applied to it and what is $v_o(t)$?
20. An ideal high-pass filter has $H(j\omega) = \begin{cases} 1 \text{ for } |\omega| > \omega_c \\ 0 \text{ for } |\omega| < \omega_c \end{cases}$. Find and plot its impulse response.
21. An ideal band-pass filter has $H(j\omega) = \begin{cases} 1 \text{ for } \omega_{c1} < |\omega| < \omega_{c2} \\ 0 \text{ for all other } \omega \end{cases}$. Find and plot its impulse response.
22. Find the percentage of normalised energy contained in the frequency band 0 to α rad/sec for a pulse $v(t) = e^{-\alpha t}$ (α is +ve real).
23. Show that $\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Re}[H(j\omega)]d\omega$ gives initial slope of step response of a circuit which has $H(j\omega)$ as its System Function.
24. Show that $H(j0)$ gives the steady-state value of step response of a circuit which has $H(j\omega)$ as its System Function.
25. Use the Fourier transform of a rectangular pulse to show that $\int_0^{\infty} \mathrm{sinc}^2(x)\,dx = 2$.
26. The energy spectral density functions for two signals $v_1(t)$ and $v_2(t)$ are identical for all ω. Does it imply that $v_1(t) = v_2(t)$ for all t? Discuss.
27. Show that $v(t)$ and $v(t-t_d)$ will have same normalised energy by employing frequency-domain reasoning.
28. The voltage across the 1H in the circuit in Fig. 14.13-4 is passed through a unity gain buffer amplifier to the 5Ω resistor. Find the total energy dissipated in 5Ω resistor if $v(t) = \delta(t)$ and the circuit was initially relaxed.

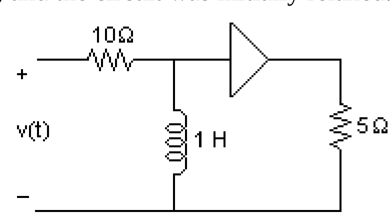

Fig. 14.13-4