Power delivered to load = $[11.12^2+11.12^2+10.68^2]\times 20$ = 7227.4 W
Power delivered by $400\angle 0^0$ V rms source = $400\times 21.08\times \cos(72.04^0)$ = 2600.3 W
Power delivered by $370\angle -120^0$ V rms source = $370\times 25.53\times \cos(-120^0+97.42^0)$ = 8772 W
Power delivered by $370\angle 120^0$ V rms source = $370\times 14.98\times \cos(120^0+102.36^0)$ = -4095.57 W

Notice the marked deviation in phase-source currents from the levels we expect from the observed line currents. We would have expected about 6 – 7 A rms in the sources considering the fact that the line currents are around 11 A rms. The circulating current due to the residual delta-loop voltage of 30 V rms makes the delta currents much larger than what they need to be in view of load power. This results in excess power being delivered by some branches forcing the remaining branch to absorb power rather than deliver it. This is why the $370\angle 120^0$ V rms source had to absorb a large quantity of power.

Of course, the delta-connected source used in this example can be of no practical use. A delta-connected source is of practical utility only if the residual voltage within the delta-loop can be kept at zero or a low value compared to source voltage.

The line voltages appearing across the load can be calculated by subtracting the voltage drop in the source impedance of each phase-source from its source voltage value. The result of this calculation is $\boldsymbol{V_{RY}} = 390\angle -0.48^0$ V rms, $\boldsymbol{V_{YB}} = 375.3\angle -121.8^0$ V rms and $\boldsymbol{V_{BR}} = 375.3\angle -120.83^0$ V rms.

## 9.5 Symmetrical Components

Analysis of balanced three-phase circuits is considerably simpler than analysis of unbalanced three-phase circuits. Three-phase symmetry exhibited by a balanced circuit makes it possible to solve the circuit employing a single-phase equivalent circuit. This prompts us to ask the question – is it possible to bring back the symmetry enjoyed by a balanced circuit into an unbalanced three-phase circuit by some means?

We realise that an unbalanced three-phase circuit can result from unbalanced three-phase sources acting on balanced loads or balanced sources acting on unbalanced loads or unbalanced three-phase sources acting on unbalanced loads. Let us tackle the first case - unbalanced three-phase sources acting on balanced loads.

### Three-Phase Circuits with Unbalanced Sources and Balanced Loads

In this case, maybe we can bring three-phase symmetry back into the circuit if we can somehow express the unbalanced three-phase source voltages/currents *as a superposition of balanced sets of three-phase voltages/currents*. Is that possible?

Specialized version of a general theorem called *Fortesque's Theorem* assures us that it is possible. If a *set* of unbalanced three-phase source functions can be expressed as a sum of balanced *sets* of three-phase source functions, then, the solution of unbalanced three-phase circuit can be obtained as a *superposition* of solution of the circuit for various balanced sets of three-phase source functions. If all the loads are balanced, each of the circuit problems that needs to be solved for applying superposition principle, will be a balanced circuit problem. Thus, symmetry can be restored to unbalanced three-phase circuits this way, *provided the unbalance is only due to sources and not due to loads.* The sets of three-phase balanced source components and possible single-phase components of an unbalanced source are called its *Symmetrical Components*.

There are three symmetrical components for an unbalanced three-phase source function. *Each symmetrical component is a set of three source functions.* The first set – called the *positive sequence component* – is a balanced three-phase set of source functions that has positive phase sequence. The second set – called the *negative sequence component* – is a balanced three-phase set of source functions that has negative phase sequence. The third set – called the *zero sequence component* – is a set of three co-phasal (*i.e.,* of same phase) single-phase source functions. *It is not a three-phase set at all.*

*Symmetrical components are denoted by the first phasor element in each set.* Thus positive sequence component is denoted by R-phase quantity or R-line quantity of the balanced three-phase source function of positive phase sequence in phasor form. Similarly, negative sequence component is denoted by R-phase quantity or R-line quantity of the balanced three-phase source function of negative phase sequence in phasor form. And, zero sequence component is denoted by one of the three co-phasal single-phase source functions.

> The sets of three-phase balanced source components and possible single-phase components that add up to form an unbalanced source are called its *Symmetrical Components*.
>
> Each symmetrical component is a set of three source functions.
>
> Symmetrical components are denoted by the first phasor element in each set.

For instance, let $200\angle\text{-}10^0$, $100\angle\text{-}50^0$ and $25\angle\text{-}30^0$ be the rms values of positive, negative and zero sequence components of some unbalanced phase voltage set. Then, $200\angle\text{-}10^0$ stands for a three-phase balanced voltage set ($200\angle\text{-}10^0$, $200\angle\text{-}130^0$, $200\angle 110^0$ ) in *RN*, *YN* and *BN* phase voltages respectively. The $100\angle\text{-}50^0$ negative sequence component stands for ($100\angle\text{-}50^0$, $100\angle 70^0$, $100\angle\text{-}170^0$) in *RN*, *YN* and *BN* phase voltages respectively. Note the phase sequence of the bracketed quantity. The zero sequence component of $25\angle\text{-}30^0$ stands for ($25\angle\text{-}30^0$, $25\angle\text{-}30^0$, $25\angle\text{-}30^0$) in *RN*, *YN* and *BN* phase voltages respectively. Then, by *Fortesque's Symmetrical Components Theorem*, the phase voltages are given by

$$\boldsymbol{V_{RN}} = 200\angle-10^0 + 100\angle-50^0 + 25\angle-30^0 = 308.8\angle-23.64^0 \text{ V rms}$$
$$\boldsymbol{V_{YN}} = 200\angle-130^0 + 100\angle 70^0 + 25\angle-30^0 = 109.4\angle-131.7^0 \text{ V rms}$$
$$\boldsymbol{V_{BN}} = 200\angle 110^0 + 100\angle-170^0 + 25\angle-30^0 = 214.66\angle 132.6^0 \text{ V rms}$$

The phase voltage set is an unbalanced one and we have expressed it as a sum of three components – one balanced positive sequence three-phase voltage, one balanced negative sequence three-phase voltage and one single-phase voltage.

We could have expressed these equations in another format too.

$$\boldsymbol{V_{RN}} = 1\times 200\angle-10^0 + 1\times 100\angle-50^0 + 1\times 25\angle-30^0$$
$$\boldsymbol{V_{YN}} = 1\angle-120^0 \times 200\angle-10^0 + 1\angle 120^0 \times 100\angle-50^0 + 1\times 25\angle-30^0$$
$$\boldsymbol{V_{BN}} = 1\angle 120^0 \times 200\angle-10^0 + 1\angle-120^0 \times 100\angle-50^0 + 1\times 25\angle-30^0$$

Let us define a constant complex number $\boldsymbol{a}$ as $\boldsymbol{a} = 1\angle 120^0$. When a phasor is multiplied by this operator, its magnitude stays unaffected, but it gets rotated in counter-clockwise direction by $120^0$. Then, $\boldsymbol{a}^2 = 1\angle 240^0 = 1\angle\text{-}120^0$ is the operator that can rotate a phasor by $120^0$ in clockwise direction. Note that $1+\boldsymbol{a}+\boldsymbol{a}^2 = 0$, $\boldsymbol{a}^2 = \boldsymbol{a}^*$, $\boldsymbol{a}^3 = 1\angle 0^0$ and $\boldsymbol{a}^4 = \boldsymbol{a}$. Now, the above equations can be expressed in terms of this operator $\boldsymbol{a}$ as

$$\boldsymbol{V_{RN}} = 1\times 200\angle-10^0 + 1\times 100\angle-50^0 + 1\times 25\angle-30^0$$
$$\boldsymbol{V_{YN}} = \boldsymbol{a}^2 \times 200\angle-10^0 + \boldsymbol{a}\times 100\angle-50^0 + 1\times 25\angle-30^0$$
$$\boldsymbol{V_{BN}} = \boldsymbol{a}\times 200\angle-10^0 + \boldsymbol{a}^2 \times 100\angle-50^0 + 1\times 25\angle-30^0$$

Now let us introduce $\boldsymbol{X_+}$, $\boldsymbol{X_-}$ and $\boldsymbol{X_0}$ as the values of positive, negative and zero sequence components (note that they are phasors) of some voltage or current instead of the three numbers- $200\angle\text{-}10^0$, $100\angle\text{-}50^0$ and $25\angle\text{-}30^0$ – we used till now. Then, the unbalanced quantities are given in terms of symmetrical components ( *i.e.*, $\boldsymbol{X_+}$, $\boldsymbol{X_-}$ and $\boldsymbol{X_0}$) by

$$\boldsymbol{X_R} = 1\times \boldsymbol{X_+} + 1\times \boldsymbol{X_-} + 1\times \boldsymbol{X_o}$$
$$\boldsymbol{X_Y} = \boldsymbol{a}^2 \times \boldsymbol{X_+} + \boldsymbol{a}\times \boldsymbol{X_-} + 1\times \boldsymbol{X_o}$$
$$\boldsymbol{X_B} = \boldsymbol{a}\times \boldsymbol{X_+} + \boldsymbol{a}^2 \times \boldsymbol{X_-} + 1\times \boldsymbol{X_o}$$

We can express this equation set in matrix form as below.

> The Symmetrical Component RYB→0+- transformation equation.

$$\begin{bmatrix}\boldsymbol{X_R}\\ \boldsymbol{X_Y}\\ \boldsymbol{X_B}\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & \boldsymbol{a}^2 & \boldsymbol{a}\\ 1 & \boldsymbol{a} & \boldsymbol{a}^2\end{bmatrix}\begin{bmatrix}\boldsymbol{X_o}\\ \boldsymbol{X_+}\\ \boldsymbol{X_-}\end{bmatrix} \quad (9.5\text{-}1)$$

$\boldsymbol{X_R}$, $\boldsymbol{X_Y}$ and $\boldsymbol{X_B}$ can represent any phase voltage, phase current, line voltage or line current phasors in an unbalanced system. The 3×3 matrix in Eqn. 9.5-1 is called the *Symmetric Transformation Matrix*. The inverse equation required for determining sequence components from phase quantities is as below.

$$\begin{bmatrix} X_o \\ X_+ \\ X_- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} X_R \\ X_Y \\ X_B \end{bmatrix} \quad (9.5\text{-}2)$$

The Symmetrical Component 0+- → RYB transformation equation.

The reader is urged to verify the matrix inversion involved.

The circuit interpretation of symmetrical components is given in Fig. 9.5-1. Here an unbalanced star-connected voltage source is resolved into its symmetrical components. Symmetrical Components theorem assures us that the source in (a) can be viewed as the composite source in (b) in which each source limb contains three sources in series – one each from each sequence component set. And superposition principle assures us that applying the source in (b) is the same as applying the three source sets shown in (c) individually.

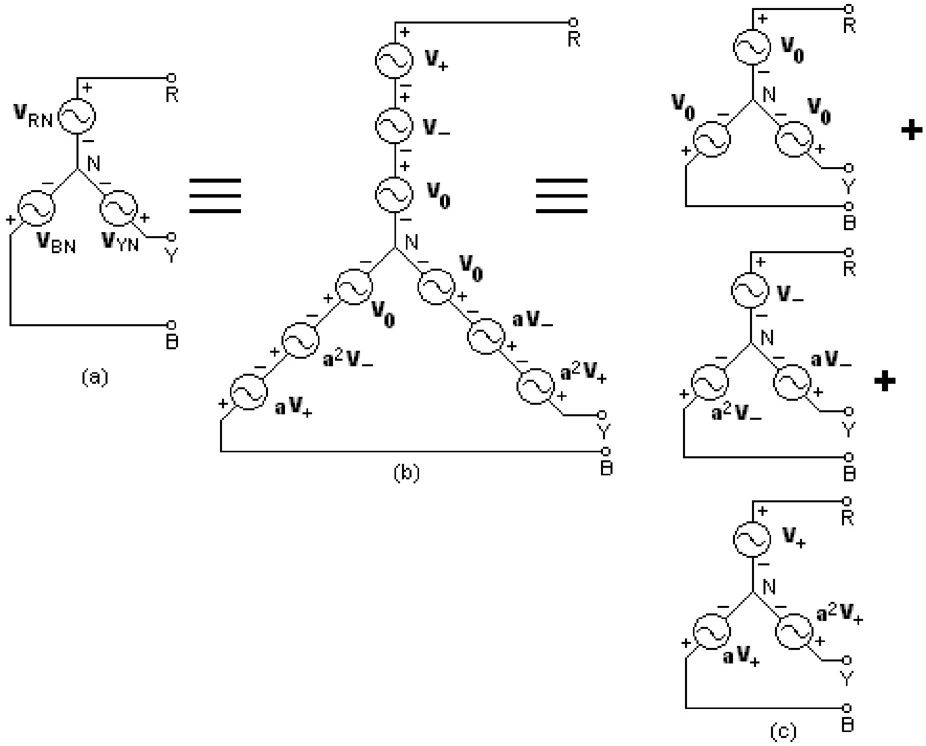


Fig. 9.5-1 Interpretation of Symmetrical Components

## The Zero Sequence Component

This component needs special attention. We get an expression for zero sequence component from Eqn. 9.5-2 as $X_o = \frac{1}{3}(X_R + X_Y + X_B)$. *Thus the zero sequence component is the average of the three three-phase quantities.*

Line voltages in any three-phase circuit, balanced or unbalanced, will add up to zero by KVL. *Therefore, line voltages in a three-phase system can not have a zero sequence content anywhere in the system.*

Line currents in a three-phase three-wire system will have to add up to zero by KCL. *Therefore, line currents in a three-phase three-wire system can not have a zero sequence content anywhere in the system.*

Phase voltages in a three-phase system *need not* add up to zero. *Hence, phase voltages can have zero sequence content.*

Line currents in a four-wire system *need not* add up to zero. The sum $I_R+I_Y+I_B$ can flow through the fourth wire (neutral wire) in the return direction. *Therefore, three-phase four-wire systems can have zero sequence content in their line currents.*

## Active Power in Sequence Components

Let the phase voltages across a *balanced three-phase load circuit* be $\boldsymbol{V_{RN}}$, $\boldsymbol{V_{YN}}$ and $\boldsymbol{V_{BN}}$ where $N$ is the neutral point in the load itself or in its Y-equivalent. Let $\boldsymbol{I_R}$, $\boldsymbol{I_Y}$ and $\boldsymbol{I_B}$ be the line currents flowing into the load. Further, let $\boldsymbol{V_0}$, $\boldsymbol{V_+}$ and $\boldsymbol{V_-}$ be the sequence components of phase voltages and $\boldsymbol{I_0}$ , $\boldsymbol{I_+}$ and $\boldsymbol{I_-}$ be the sequence components of line currents.

Then the total active power that flows into the load,

$$P = \text{Re}[\boldsymbol{V_{RN}}\boldsymbol{I_R}^*] + \text{Re}[\boldsymbol{V_{YN}}\boldsymbol{I_Y}^*] + \text{Re}[\boldsymbol{V_{BN}}\boldsymbol{I_B}^*]$$
$$= \text{Re}[\boldsymbol{V_{RN}}\boldsymbol{I_R}^* + \boldsymbol{V_{YN}}\boldsymbol{I_Y}^* + \boldsymbol{V_{BN}}\boldsymbol{I_B}^*]$$

Let us define four column vectors as below.

$$\boldsymbol{V_{ryb}} = \begin{bmatrix} \boldsymbol{V_{RN}} \\ \boldsymbol{V_{YN}} \\ \boldsymbol{V_{BN}} \end{bmatrix}; \ \boldsymbol{I_{ryb}} = \begin{bmatrix} \boldsymbol{I_R} \\ \boldsymbol{I_Y} \\ \boldsymbol{I_B} \end{bmatrix}; \boldsymbol{V_{o+-}} = \begin{bmatrix} \boldsymbol{V_o} \\ \boldsymbol{V_+} \\ \boldsymbol{V_-} \end{bmatrix}; \ \boldsymbol{I_{o+-}} = \begin{bmatrix} \boldsymbol{I_o} \\ \boldsymbol{I_+} \\ \boldsymbol{I_-} \end{bmatrix}$$

Then the power equation can be expressed as $P = \text{Re}[\boldsymbol{V_{ryb}^t}\,\boldsymbol{I_{ryb}^*}]$ where the superscripts $\boldsymbol{t}$ indicates matrix transpose operation and ***** indicates complex conjugation operation.

Eqn. 9.5-1 expresses the three-phase quantities in terms of its sequence components. It is reproduced below.

$$\begin{bmatrix} \boldsymbol{X_R} \\ \boldsymbol{X_Y} \\ \boldsymbol{X_B} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a^2} & \boldsymbol{a} \\ 1 & \boldsymbol{a} & \boldsymbol{a^2} \end{bmatrix} \begin{bmatrix} \boldsymbol{X_o} \\ \boldsymbol{X_+} \\ \boldsymbol{X_-} \end{bmatrix}$$

Using this equation, we write the column vector $\boldsymbol{V_{ryb}}$ in terms of the column vector $\boldsymbol{V_{0+-}}$ and write the column vector $\boldsymbol{I_{ryb}}$ in terms of the column vector $\boldsymbol{I_{0+-}}$ as below.

$$\boldsymbol{V_{ryb}} = \boldsymbol{A}\boldsymbol{V_{o+-}} \text{ and } \boldsymbol{I_{ryb}} = \boldsymbol{A}\,\boldsymbol{I_{o+-}} \text{ where } \boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a^2} & \boldsymbol{a} \\ 1 & \boldsymbol{a} & \boldsymbol{a^2} \end{bmatrix}$$

Then,

$$P = \text{Re}[\boldsymbol{V_{ryb}^t}\,\boldsymbol{I_{ryb}^*}] = \text{Re}[(\boldsymbol{A}\boldsymbol{V_{o+-}})^t(\boldsymbol{A}\boldsymbol{I_{o+-}})^*] = \text{Re}[\boldsymbol{V_{o+-}}^t\boldsymbol{A}^t\boldsymbol{A}^*\boldsymbol{I_{o+-}}^*]$$

$$\boldsymbol{A}^t = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a^2} & \boldsymbol{a} \\ 1 & \boldsymbol{a} & \boldsymbol{a^2} \end{bmatrix} \text{ and } \boldsymbol{A}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & (\boldsymbol{a^2})^* & \boldsymbol{a}^* \\ 1 & \boldsymbol{a}^* & (\boldsymbol{a^2})^* \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a} & \boldsymbol{a^2} \\ 1 & \boldsymbol{a^2} & \boldsymbol{a} \end{bmatrix}$$

Therefore, $\boldsymbol{A^t A^*}$ is identity matrix of 3x3 order. Hence,

$$\begin{aligned} P &= \text{Re}[\boldsymbol{V_{o+-}}\boldsymbol{I_{o+-}}^*] = \text{Re}[\boldsymbol{V_o}\boldsymbol{I_o}^* + \boldsymbol{V_+}\boldsymbol{I_+}^* + \boldsymbol{V_-}\boldsymbol{I_-}^*] \\ &= \text{Re}[\boldsymbol{V_o}\boldsymbol{I_o}^*] + \text{Re}[\boldsymbol{V_+}\boldsymbol{I_+}^*] + \text{Re}[\boldsymbol{V_-}\boldsymbol{I_-}^*] \end{aligned} \tag{9.5-3}$$

Thus, we observe that (i) active power carried by sequence components obey superposition principle and that (ii) positive sequence component of voltage can send active power *only through* positive sequence component of current; negative sequence component of voltage can send active power *only through* negative sequence component of current and zero sequence component of voltage can send active power *only through* zero sequence component of current.

> Sequence decoupling
>
> Balanced impedance will have negative sequence current in it only if there is negative sequence voltage applied to it.
>
> This is so since balanced impedance can produce only negative sequence voltage drop when a negative sequence current flows through it.
>
> That is, *balanced impedance is incapable of bringing about a sequence conversion involving translation of positive sequence current into negative sequence voltage and vice-versa*.
>
> This is called *sequence-decoupling property* of balanced three-phase impedances.

## Three-Phase Circuits with Balanced Sources and Unbalanced Loads

Three-phase sources, at least at generation and transmission level, are more or less balanced. They may show considerable unbalance at LT distribution level. However, three-phase circuit unbalance is usually the result of unbalanced loads rather than unbalanced sources. Symmetrical components can be used for analysing such circuits too. However, we do not take up such analysis since it is somewhat beyond the scope of an introductory text like this one. Hence, we close our discussion on symmetrical components after covering two salient points in this sub-section.

The first point is that an unbalanced load does not possess *sequence-decoupling property*. Rather, *it creates sequence coupling*. Consider a balanced three-phase voltage source supplying an unbalanced three-phase load. Since the voltage source is balanced, it has no negative and zero sequence components. It is a purely positive sequence voltage source. However, the load will draw unbalanced currents since it is unbalanced. We know that unbalanced current set will contain negative sequence component. Thus a positive sequence voltage source has produced a negative sequence current in the circuit. Moreover, the voltage drop produced across the load impedance due to the negative sequence current must be a positive sequence voltage since there is only positive sequence component in the source voltage to meet the KVL requirement. Therefore, negative sequence current succeeds in producing a positive sequence voltage drop in the case of an unbalanced load. Therefore, *an unbalanced load produces sequence coupling between positive sequence components and negative sequence components.*

The second observation we want to make is regarding active power. We observed in the last sub-section that a negative sequence current flowing through a positive sequence voltage could not carry active power. Hence, the negative sequence current drawn by an unbalanced three-phase load from a balanced three-phase source is a wasteful current that ties up equipment capacity and increases system losses without carrying productive power. We had met with another such current before – the reactive current component. Thus, both the reactive component of positive sequence current and the entire negative sequence current drawn by an unbalanced three-phase load from a balanced supply are unproductive in effect and detrimental to the system. *We conclude that the most energy-efficient way to draw active power from a balanced three-phase source is to draw it through balanced three-phase currents at unity power factor.* Symmetrical components afford us the required information to evaluate the effectiveness of current in carrying power in unbalanced loading of the system equipment. We can solve the unbalanced circuit problem by applying mesh or nodal analysis and obtain the symmetrical components of currents to evaluate how bad the current utilization is.

Thus, with this introductory coverage on symmetrical components, we can now apply sequence components (i) to solve circuits with *unbalanced sources* and *balanced loads* and (ii) to evaluate the effectiveness of current flow in carrying active power in circuits with *balanced sources* and *unbalanced loads*. There is much more to symmetrical components. Power System Engineers go deep into symmetrical components in analysing power systems under fault condition.

**Example : 9.5-1**

An unbalanced 4-wire system is shown in Fig. 9.5-2. (i) Find the symmetrical components of the source phase voltages. (ii) Determine the line voltages at the source and verify that line voltage does not contain zero sequence component. (iii) Determine the line currents and neutral current by using symmetrical components. (iv) Find the load neutral voltage with respect to earth. (v) Find the phase voltages and line voltages across the load by symmetrical components. (vi) Find the total power delivered by source and delivered to load using symmetrical components.

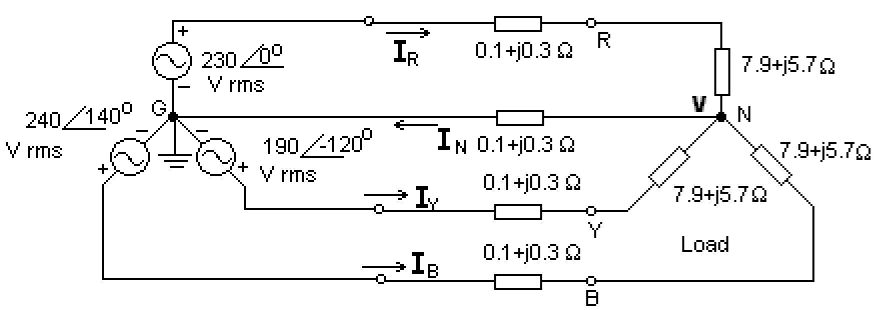


Fig. 9.5-2 Unbalanced Three-Phase Circuit for Example : 9.5-1

**Solution**

(i) $\begin{bmatrix} \boldsymbol{V_{so}} \\ \boldsymbol{V_{s+}} \\ \boldsymbol{V_{s-}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a} & \boldsymbol{a}^2 \\ 1 & \boldsymbol{a}^2 & \boldsymbol{a} \end{bmatrix}\begin{bmatrix} \boldsymbol{V_{sRG}} \\ \boldsymbol{V_{sYG}} \\ \boldsymbol{V_{sBG}} \end{bmatrix}$. The subscript '***s***' stands for source quantities. Substituting $\boldsymbol{a} = 1\angle 120^0$, $\boldsymbol{V_{sRN}} = 230\angle 0^0$, $\boldsymbol{V_{sYN}} = 190\angle\text{-}120^0$ and $\boldsymbol{V_{sBN}} = 240\angle 140^0$,

$$\begin{bmatrix} V_{so} \\ V_{s+} \\ V_{s-} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1\angle 120^0 & 1\angle -120^0 \\ 1 & 1\angle -120^0 & 1\angle 120^0 \end{bmatrix}\begin{bmatrix} 230\angle 0^0 \\ 190\angle -120^0 \\ 240\angle 140^0 \end{bmatrix} = \begin{bmatrix} 16.64\angle -168.12^0 \\ 216.91\angle 7.25^0 \\ 39.25\angle -37.58^0 \end{bmatrix} \text{V rms}$$

(ii) Line voltages are obtained as below.

$$V_{sRY} = V_{sRG} - V_{sYG} = 230\angle 0^0 - 190\angle -120^0$$
$$= 325 + j164.54 = 364.28\angle 26.85^0 \text{ V rms}$$
$$V_{sYB} = V_{sYG} - V_{sBG} = 190\angle -120^0 - 240\angle 140^0$$
$$= 88.85 - j318.81 = 330.96\angle \text{-}74.43^0 \text{ V rms}$$
$$V_{sBR} = V_{sBG} - V_{sRG} = 240\angle 140^0 - 230\angle 0^0$$
$$= -413.85 + j154.27 = 441.67\angle 159.56^0 \text{ V rms}$$
$$V_{sRY} + V_{sYB} + V_{sBR} = (325 + j164.54) + (88.85 - j318.81) + (-413.85 + j154.27)$$
$$= 0 + j0$$

Therefore, zero sequence component in line voltage is zero.

(iii) The circuit is solved for the three symmetrical components by applying superposition principle. We apply the positive sequence voltage source component first and obtain the positive sequence component of line currents and neutral current. We note that the connection impedances and load impedances are balanced. Therefore we can use single-phase equivalent circuit for obtaining currents. The single-phase equivalent circuit is shown in Fig. 9.5-3. The source neutral and load neutral are at the same potential and hence the neutral impedance of 0.1+$j$0.3 Ω will not appear in the single-phase equivalent circuit for positive sequence component.

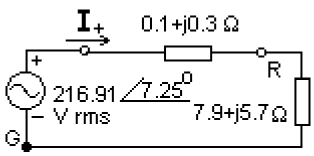

Fig. 9.5-3 Single-Phase Equivalent Circuit of the Circuit in Fig. 9.5-2 for Positive Sequence Component

Therefore, $I_+$ = 216.91∠7.25$^0$ ÷ (8+$j$6) = 21.69∠-29.62$^0$ A rms.

The single-phase equivalent circuit for negative sequence input is shown in Fig. 9.5-4. Passive balanced impedance can not differentiate between positive phase sequence and negative phase sequence. However, this is not true in the case of all electrical equipment. AC motors can distinguish between the two and hence the equivalent circuits of motors will be different for the two phase sequences.

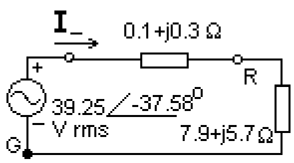

Fig. 9.5-4 Single-Phase Equivalent Circuit of the Circuit in Fig. 9.5-2 for Negative Sequence Component

The load circuit is balanced and negative sequence component is a balanced three-phase component. Therefore neutral potential at source and load will be the same and the neutral impedance will not appear in the single-phase equivalent circuit for negative sequence input too.

Therefore, $I_-$ = 39.25∠-37.58$^0$ ÷ (8+$j$6) = 3.925∠-74.45$^0$ A rms.

The circuit to be solved with zero sequence input is shown in Fig. 9.5-5.

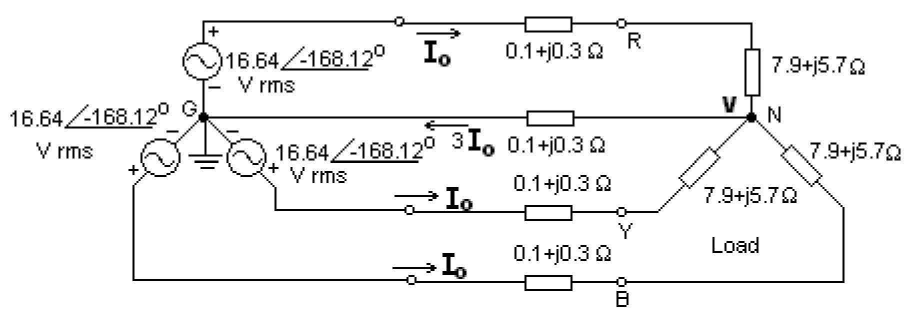

Fig. 9.5-5 Circuit with only Zero Sequence Component of Input Voltage Acting in the Circuit in Fig. 9.5-2

**Note that the effective value of neutral line impedance in limiting the zero sequence current is three times its actual value due to $3I_0$ flowing in it.**

Note that all the three voltage sources have the same phase and hence all the three lines R,Y and B will carry co-phasal currents in the same direction. Therefore the neutral return current will be 3$I_0$ . Applying KVL in any one mesh, we get,

[(8+$j$6)+3×(0.1+$j$0.3)] ×$I_0$ = 16.64∠-168.12$^0$ ⇒ $I_0$ = 1.542∠152.14$^0$ A rms

Now the line currents can be obtained by applying symmetrical component transformation equation.

$$\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_o \\ I_+ \\ I_- \end{bmatrix}$$

The result will be, $I_R$ = 23.106∠-36.62$^0$ A rms, $I_Y$ = 18.86∠-156.75$^0$ A rms and $I_B$ = 23.984∠-102.78$^0$ A rms.

Neutral current can arise only from zero sequence component of source voltage since the remaining two components are balanced three-phase components and the load circuit is balanced. Hence neutral current = 3$I_0$ = 4.63∠152.14$^0$ A rms.

(iv) Load neutral voltage with respect to earth = neutral current × neutral impedance, since the source neutral is earthed. The value is = 4.63∠152.14$^0$ × (0.1+$j$0.3) = 1.463 ∠-136.3$^0$ V rms.

(v) The symmetrical components of load phase voltages can be obtained as

$$\begin{bmatrix} V_{lo} \\ V_{l+} \\ V_{l-} \end{bmatrix} = \begin{bmatrix} 7.9+j5.7 & 0 & 0 \\ 0 & 7.9+j5.7 & 0 \\ 0 & 0 & 7.9+j5.7 \end{bmatrix} \begin{bmatrix} I_o \\ I_+ \\ I_- \end{bmatrix}$$

The values are $V_{l0}$ = 15.018∠-172.05$^0$ V rms, $V_{l+}$ = 211.305∠6.2$^0$ V rms and $V_{l-}$ = 38.237∠-38.64$^0$ V rms.

Now the load phase voltages can be determined by symmetrical components transformation.

$$\begin{bmatrix} V_{lRN} \\ V_{lYN} \\ V_{lBN} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{lo} \\ V_{l+} \\ V_{l-} \end{bmatrix}$$

The values obtained on substitution of sequence component values are,

$V_{lRN}$ = 225.09∠-0.81$^0$ V rms, $V_{lYN}$ =183.73∠-120.93$^0$ V rms and $V_{lBN}$ =233.65∠138.54$^0$ V rms.

The load line voltages are found as

$$V_{lRY} = V_{lRN} - V_{lYN} = 225.09\angle -0.81^0 - 183.73\angle -120.93^0 = 354.87\angle 25.8^0 \text{ V rms}$$
$$V_{lYB} = V_{lYN} - V_{lBN} = 183.73\angle -120.93^0 - 233.65\angle 138.54^0 = 322.41\angle -75.5^0 \text{ V rms}$$
$$V_{lBR} = V_{lBN} - V_{lRN} = 233.65\angle 138.54^0 - 225.09\angle -0.81^0 = 430.26\angle 158.5^0 \text{ V rms}$$

(vi) The values of sequence components at source end are

$V_{so} = 16.64\angle -168.12^0$ V rms; $V_{s+} = 216.91\angle 7.25^0$ V rms; $V_{s-} = 39.25\angle -37.58^0$ V rms and $I_0$ = 1.542∠152.14$^0$ A rms ; $I_+$ = 21.69∠-29.62$^0$ A rms ; $I_-$ = 3.925∠-74.45$^0$ A rms.

∴Power delivered through zero sequence components

= 16.64×1.542×cos(-168.12$^0$-152.14$^0$) = 19.73 W per phase

∴Power delivered through positive sequence components

= 216.91×21.69×cos(7.25$^0$+29.62$^0$) = 3763.9 W per phase

∴Power delivered through negative sequence components

= 39.25×3.925×cos(-37.58$^0$+74.45$^0$) = 123.3 W per phase

∴Total power delivered by the source = 3×(19.73+3763.9+123.3)W = 11.72 kW.

The values of sequence components at load end are,

$V_{l0}$ = 15.02∠-172.1$^0$ V rms, $V_{l+}$ = 211.31∠6.2$^0$ V rms and $V_{l-}$ = 38.24∠-38.6$^0$ V rms and $I_0$ = 1.54∠152.14$^0$ A rms, $I_+$ = 21.69∠-29.6$^0$ A rms, $I_-$ = 3.93∠-74.5$^0$ A rms.

∴Power delivered to the load through zero sequence components

= 15.018×1.542×cos(-172.05$^0$ -152.14$^0$) = 18.8 W per phase

∴Power delivered to the load through positive sequence components

= 211.305×21.69×cos(6.2$^0$ +29.62$^0$) = 3716.9 W per phase

∴Power delivered to the load through negative sequence components

= 38.237×3.925×cos(-38.64$^0$+74.45$^0$) = 121.7 W per phase

∴Total power delivered to the load = 3×(18.8+3716.9+121.7) W = 11.572 kW

## Example : 9.5-2

Single-phasing of a three-phase load leads to an extreme case of unbalanced operation in practice. An undetected open in the wiring path in one phase or a fuse blowing in a phase that leads to this kind of operation of three-phase loads in Industries. It is so common that specific measures in the form of a single-phasing prevention relay is incorporated in the control gear of high-power induction motor drives in the Industry.

The circuit in Fig. 9.5-6 shows a balanced three-phase induction motor running from a balanced supply. The balanced operation of this circuit was studied in Example : 9.3-2. Assume that the line goes open in B-phase leading to single-phased operation of the motor. Find (i) line currents and sequence components of line currents (ii) currents in the three motor windings (iii) active and reactive power delivered to the motor (iv) Apparent power and power factor of the source.

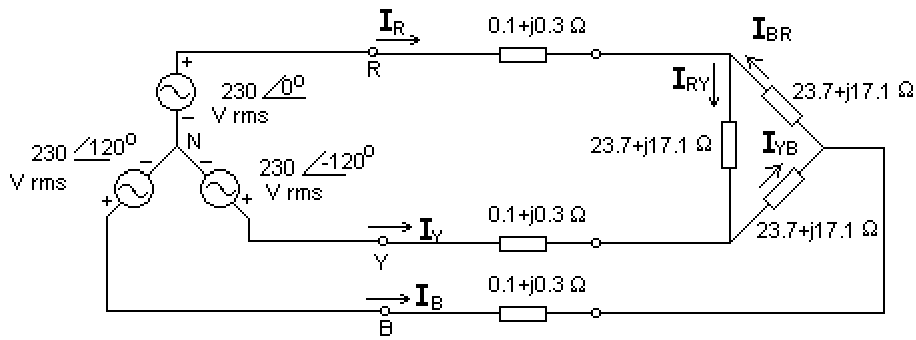


Fig. 9.5-6 Circuit for Example : 9.5-2

### Solution

This circuit was analysed in Example : 9.3-2 for balanced operation. The following results were obtained.

Line current (R) = $23.09\angle -36.9^0$ A rms, Load phase voltage (R) = $224.9\angle -1.10$ V rms and Load line voltage (RY) = $389.56\angle 28.9^0$ V rms. The delta branch currents had a rms value of 13.33 A.

Load active power = 12.64 kW, load reactive power = 9.11 kVAr and source apparent power = 16 kVA.

We replace the balanced delta-connected load impedance by a balanced star-connected impedance first to get a Y-connected equivalent circuit for the circuit in Fig. 9.5-6. This Y-connected equivalent circuit with the break in B-line is shown in Fig. 9.5-7. Note that there is only one current variable in the circuit now and that current phasor is marked as $\boldsymbol{I}$.

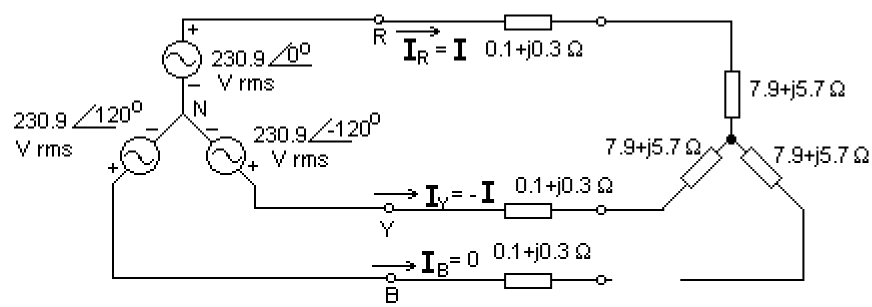


Fig. 9.5-7 The Y-connected Equivalent Circuit of Circuit in Fig. 9.5-6 with B-line Open

(i) The current phasor $\boldsymbol{I}$ can be obtained by writing the KVL equation in the only mesh that is present in the circuit.

$\boldsymbol{I} = [230.9\angle 0^0 - 230.9\angle -120^0] \div [16+j12] = 400\angle 30^0 \div 20\angle 36.87^0 = 20\angle -6.87^0$ A rms

Therefore, the line currents are $\boldsymbol{I_R} = 20\angle -6.87^0$ A rms, $\boldsymbol{I_Y} = -20\angle -6.87^0$ A rms and $\boldsymbol{I_B} = 0$.

The symmetrical components of line current is found by using the equation

$$\begin{bmatrix} \boldsymbol{I_o} \\ \boldsymbol{I_+} \\ \boldsymbol{I_-} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a} & \boldsymbol{a^2} \\ 1 & \boldsymbol{a^2} & \boldsymbol{a} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_R} \\ \boldsymbol{I_Y} \\ \boldsymbol{I_B} \end{bmatrix}.$$

The sequence components are $\boldsymbol{I_0}$ = 0, $\boldsymbol{I_+}$ =11.545∠-36.87$^0$ A rms and $\boldsymbol{I_-}$ = 11.545∠23.13$^0$ A rms. Observe that the positive sequence and negative sequence components of current have equal magnitude.

(ii) The current ***I*** divide in two paths in the delta-connected windings. All the windings have equal impedances. Therefore the current is shared in 2:1 ratio in RY winding and other two windings in series. The winding currents are 13.33 A rms, 6.67 A rms and 6.67 A rms.

(iii) Active power delivered to motor = Active power delivered by source – power dissipated in connection impedance= 400×20×cos(30$^0$-(-6.87$^0$))-20$^2$×0.1×2 W = 6.32 kW

Reactive power delivered to motor = *Q* delivered by source – *Q* absorbed by connection impedance = 400×20×sin(30$^0$-(-6.87$^0$)) - 20$^2$×0.3×2 VAr = 4.56 kVAr.

(iv) Apparent power of source = 230.9×20+230.9×20+230.9×0 VA = 9.236 kVA

Power factor = 6.4/9.236 = 0.693

## Example : 9.5-3

Convert the delta-connected unbalanced voltage source with source impedance in Fig. 9.5-8 into its Y-connected equivalent.

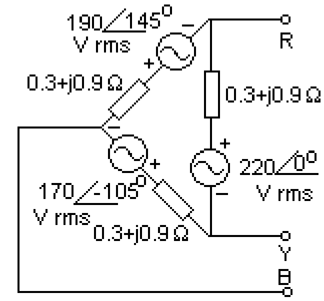


Fig. 9.5-8 Delta-connected Unbalanced Source in Example : 9.5-3

### Solution

Let the three source voltage phasors be identified as $\boldsymbol{V_1}$ , $\boldsymbol{V_2}$ and $\boldsymbol{V_3}$ . Let the source impedance be identified as ***Z***. We convert the delta branches into current sources in parallel with impedance form as shown in Fig. 9.5-9.

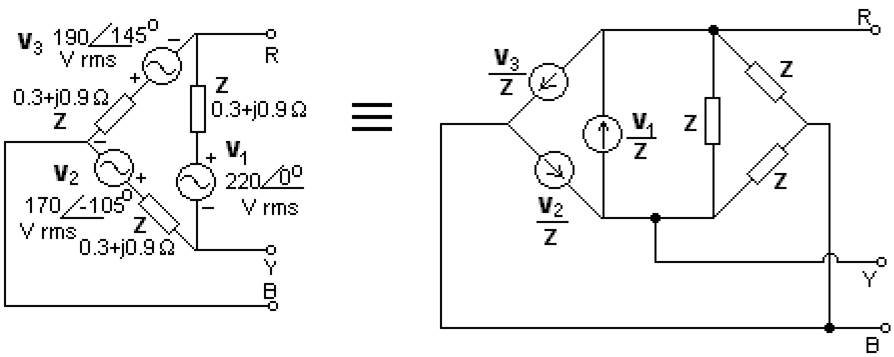


Fig. 9.5-9 Voltage Source to Current Source Transformation Applied to the Source in Fig. 9.5-8

The delta-connected unbalanced current source is now resolved in terms of its sequence components as shown in Fig. 9.5-10 where $\boldsymbol{I_+} = \boldsymbol{V_+}/\boldsymbol{Z}$ , $\boldsymbol{I_-} = \boldsymbol{V_-}/\boldsymbol{Z}$ and $\boldsymbol{I_0} = \boldsymbol{V_0}/\boldsymbol{Z}$ .

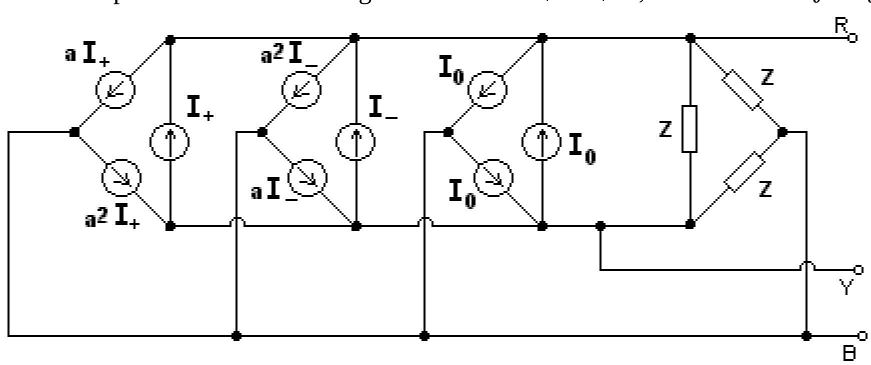


Fig. 9.5-10 Resolution of Three-Phase Current Source in Fig. 9.5-9
in terms of its Sequence Components

The first two delta-connected current sources are three-phase balanced sources and have Y-equivalents. Their Y-equivalent is obtained by finding out the first line current delivered by each.

The positive sequence sources deliver $\boldsymbol{I}_+$ - $\boldsymbol{aI}_+$ in to R-line. The star-equivalent must have this as the source function in R-phase. (1-$\boldsymbol{a}$) = $\sqrt{3}\angle 30^0$. Therefore the star-equivalent will have a positive sequence current source of value of $\sqrt{3}\angle 30^0\,\boldsymbol{I}_+$.

The negative sequence sources deliver $\boldsymbol{I}_-$ - $\boldsymbol{a}^2\,\boldsymbol{I}_-$ in to R-line. The star-equivalent must have this as the source function in R-phase. (1-$\boldsymbol{a}^2$ ) = $\sqrt{3}\angle -30^0$. Therefore the star-equivalent will have a positive sequence current source of value of $\sqrt{3}\angle -30^0\,\boldsymbol{I}_-$.

The zero sequence current inside the delta circulates within it and can not come out. Therefore the Y-equivalent has no zero sequence current source. This is consistent with the fact that the line currents in a three-wire system can not have zero sequence content.

Therefore the Y-equivalent of the circuit in Fig. 9.5-10 is as shown in circuit (a) of Fig. 9.5-11. Note that the Δ-connected impedance is also converted into equivalent star.

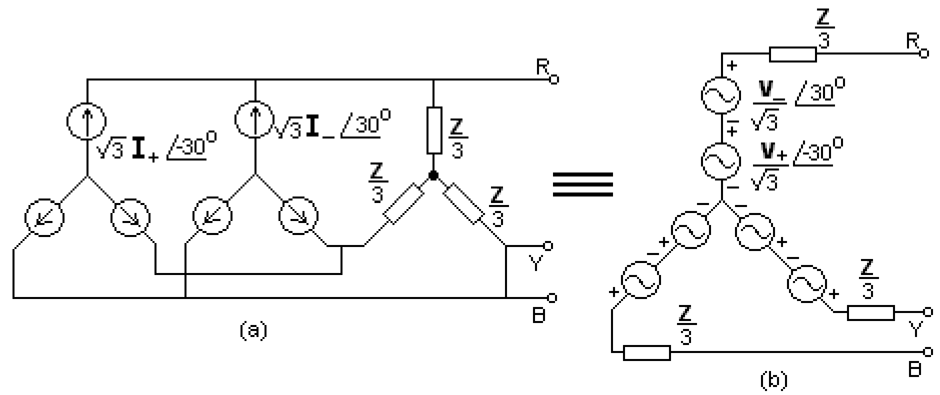


Fig. 9.5-11 Y-equivalent of Circuit in Fig. 9.5-10

Now all the three three-phase Y-connected sub-circuits in circuit (a) are balanced circuits and hence the neutrals will be at the same potential. Therefore, they can be joined together. Then, the resulting parallel connection of two current sources and impedance in each phase can be converted into two voltage sources in series with the same impedance. The last step is to substitute for $\boldsymbol{I}_+$ and $\boldsymbol{I}_-$ in terms of sequence components of delta phase-source voltages. The resulting Y-connected equivalent circuit is shown in circuit (b) of Fig. 9.5-11.

Now we have determined the positive sequence and negative sequence components of phase voltages of the Y-connected equivalent. They are $\frac{\boldsymbol{V}_+}{\sqrt{3}}\angle -30^0$ and $\frac{\boldsymbol{V}_-}{\sqrt{3}}\angle 30^0$ respectively. The zero sequence component is zero. Note that there is non-zero zero sequence content in delta-connected sources.

$$\begin{bmatrix}\boldsymbol{V_o}\\ \boldsymbol{V}_+\\ \boldsymbol{V}_-\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \boldsymbol{a} & \boldsymbol{a}^2\\ 1 & \boldsymbol{a}^2 & \boldsymbol{a}\end{bmatrix}\begin{bmatrix}\boldsymbol{V_1}\\ \boldsymbol{V_2}\\ \boldsymbol{V_3}\end{bmatrix}$$

Substituting for $\boldsymbol{V_1}$, $\boldsymbol{V_2}$, $\boldsymbol{V_3}$ and $\boldsymbol{a}$ , we get $\boldsymbol{V_0}$ = 19.62∠-69.76$^0$ V rms, $\boldsymbol{V}_+$ =190.04∠12.6$^0$ V rms, and $\boldsymbol{V}_-$ =36.051∠-39.69$^0$ V rms.

The star equivalent phase voltage are obtained by

$$\begin{bmatrix}\boldsymbol{V_{RN}}\\ \boldsymbol{V_{YN}}\\ \boldsymbol{V_{BN}}\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\\ 1 & \boldsymbol{a}^2 & \boldsymbol{a}\\ 1 & \boldsymbol{a} & \boldsymbol{a}^2\end{bmatrix}\begin{bmatrix}\frac{1\angle -30^0}{\sqrt{3}}\boldsymbol{V}_+\\ \frac{1\angle 30^0}{\sqrt{3}}\boldsymbol{V}_-\\ 0\end{bmatrix}=\begin{bmatrix}130.38\angle\text{-}16.17^0\\ 103.63\angle\text{-}148.11^0\\ 98.38\angle 112.23^0\end{bmatrix}\text{ V rms}.$$