satisfying Ohm's law on an instant to instant basis. If $v(t)$ is the electrostatic potential difference across the resistance and $i(t)$ is the current entering the higher potential terminal, then $v(t) = i(t)\,R$.

1.3 Two-terminal Capacitance

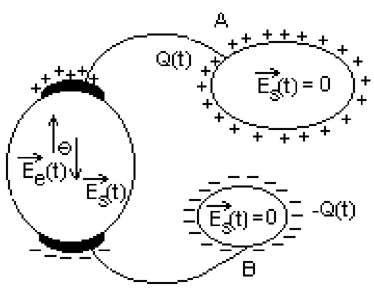

Fig. 1.3-1 A time-varying e.m.f source with two electrodes

Consider the electrical system shown in Fig. 1.3-1. A source of time-varying electromotive force is connected to a pair of metallic electrodes A and B. We assume that the connecting wires are of infinite conductivity and near-zero cross-section. Further, we assume that the metallic electrodes are made of material with infinite conductivity. Therefore the electrostatic field inside the two electrodes will be zero at all t even when there is current flow in the electrode material.

The total surface charge distributed on the conducting surfaces in the system has two components – the charge distributed on the source terminal and the charge distributed on the electrode. The charge distributed on the connecting wire is negligible since the wire is assumed to be very thin.

The surface charges on the source terminals and electrodes will assume suitable magnitudes and suitable distributions such that (i) the non-electrostatic field in the source is cancelled by the electrostatic field created by the charge distributions on an instant to instant basis everywhere within and (ii) the electrostatic field everywhere inside the connecting wires and electrodes is zero at all time. Thus, $Q(t)$, the total charge distributed in the electrode and the manner in which it is distributed will depend on $\vec{E}_e(x, y, z, t)$ of the source, the spatial geometry of the entire system and material/medium dielectric properties. Therefore, $Q(t)$ will change if the source is moved without affecting the relative position of electrodes. The voltage between the electrodes A and B - *i.e.*, $V_{AB}(t)$ – will be equal to the electromotive force always; but the charge stored in the electrode system will vary with the spatial position of the source. Thus a unique ratio between $Q(t)$ and $V_{AB}(t)$ will exist only for a particular spatial arrangement of source and electrodes. The ratio will change with the position of source and can not be called a property of electrode arrangement alone.

The ratio between charge and voltage of a pair of electrodes depends on relative position of electrode system with respect to other components.

All components in an electrical system will have static charge distributions at their terminals and on their surfaces. The electrostatic field at a point is the superposition of fields created by all these charge distributions. Thus, the voltage across terminals of one component will be decided by the work done in carrying a unit positive charge across the terminal pair against an electrostatic force that is decided by the static charge distributions in the entire electrical system. Thus, a simple ratio of the voltage across terminals of one component to the value of charge distributed at its terminals and surface can not be defined in general.

Now we introduce certain assumptions so that we can ascribe the ratio $Q(t)/V_{AB}(t)$ to the electrode pair A and B without any reference to the position of other elements in the system. We assume that the distance between electrodes and the physical dimensions of the two-electrode system are very small compared to the distance between the two-electrode system and other components in the electrical system. [The reader may think of a parallel plate capacitor of large capacitance value and wonder how such a capacitor can satisfy this requirement. That is precisely why a parallel plate capacitor is found only in the pages of textbooks. A practical 'parallel plate capacitor' has two aluminium foils of large length rolled into a tight cylinder shape with a pair of dielectric films between them. Such an assembly of a pair of electrodes will satisfy the assumption stated above.]

Assumption required for defining a two-terminal capacitor

Positive and negative charge distributions of equal magnitude kept close to each other will produce only negligible electrostatic field at distant points. Therefore, the charge distribution on a pair of electrodes that satisfy the assumption stated above will not affect the electrostatic field at the locations where other circuit components are located. And, charge distributions on other circuit components will not affect the electrostatic field at the location where this electrode pair is located. Therefore the ratio of charge stored in the electrodes to voltage between the electrodes will depend only on the geometry of the electrode system and dielectric properties of the medium involved.

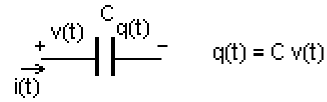

Fig. 1.3-2 A two-terminal capacitor

This unique and constant ratio associated with an electrode pair is defined as its capacitance value and the electrode system that satisfies the assumptions explained above is termed as a *two-terminal capacitor*. The magnitude of charge stored in one of the electrodes in a *linear capacitor* is proportional to the voltage across it. The symbol and variable assignment of a two-terminal capacitance is shown in Fig. 1.3-2.

In fact, Circuit Theory extends the assumption of '*locally confined stationary electrostatic field*' to all components in the circuit. It assumes that the electrostatic field created by the charge distribution residing on a particular component (remember that there is no charge distribution on wires; they are of near-zero cross-section. Therefore, charge distributions can be ascribed to components uniquely) is significant only in the vicinity of that component and is negligible at the locations of other components. This makes the electrostatic field around a component a function of its own charge distribution alone. Therefore, the potential difference across terminals of one component will be proportional to the charge distributed on it. Thus assumption of '*locally confined stationary electrostatic field*' amounts to neglecting electrostatic coupling between various components. With this assumption, the voltage across a component becomes proportional to the total charge distributed on its terminals and conducting surfaces. The proportionality constant depends on the geometry of the component as well as on material dielectric properties. The fact that there has to be a certain amount of charge distributed on the surface of a component for a voltage difference to exist between its terminals is equivalently described as *the capacitive effect* present in the component. *Thus every electrical component has a capacitive effect inherent in it.*

The assumption of '*locally confined electrostatic fields*' in Circuit Theory

Therefore, a piece of conductor too has a capacitive effect associated with it. We ignored the current component that is required to support a time-varying charge distribution across a resistance in the previous section (Section 1.2) in order to define a two-terminal resistance. This is equivalent to neglecting the capacitive effect that is invariably present in the resistance. There is no pure resistance element in practice. All resistors come with a capacitive effect. However, if the capacitance that is present across a resistor draws only negligible current in a given circumstance, then, it may be modeled by a two-terminal resistance.

The capacitance that is present across a two-terminal resistance is called the *parasitic capacitance* associated with it. The adjective '*parasitic*' gives us an impression that it is some second-order effect that has only nuisance value. That is not true – it arises out of the charge distribution that is required to make conduction possible in the resistance. Without this parasitic capacitance the resistor will not carry any current at all!

The relation between the charge stored in a capacitor and voltage across it is given by $q(t) = Cv(t)$. *C*, the capacitance value has Coulomb per Volt as its unit. This unit is given a special name – 'Farad'. One Farad is too large a value for capacitance in practice. Practical capacitors have capacitance value ranging from few pFs (1 pF = 10^{-12} F) to few thousand μFs (1μF = 10^{-6} F). The value of *C* is a constant if the geometry of capacitor does not change with time and the material that is used as the dielectric between the metallic electrodes is linear, homogeneous and isotropic. If the value of *C* is a constant, it is called a *linear capacitor*.

The current that has to flow into the positively charged electrode of the capacitor is given by rate of change of the charge residing in that electrode. Therefore, the voltage across a linear capacitor is related to the current flowing into the positive electrode as below.

$$\begin{aligned} q(t) &= Cv(t) \\ i(t) &= C\frac{dv(t)}{dt} \\ v(t) &= \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = \frac{1}{C}v(0) + \frac{1}{C}\int_{0}^{t} i(t)\,dt \end{aligned} \tag{1.3-1}$$

Voltage-Current relations of a linear capacitor. *i*(*t*) flows into the positive polarity of *v*(*t*).

The current through a capacitor depends on the first derivative of voltage appearing across it. Therefore, the current flow through the parasitic capacitance that is inevitably present across any electrical element can be neglected in the circuit model for that element only if the rate of change of electrical quantities involved in the circuit is small enough. Thus, a two-terminal resistance will model a piece of conducting substance with sufficient accuracy only if the frequency of voltage and current variables in the circuit is sufficiently small.

We have seen that there is no purely resistive two-terminal element in the physical world. A parasitic capacitance always goes along with a resistance. However, is there a pure two-terminal capacitor in real world?

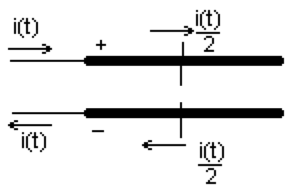

Fig. 1.3-3 Pertaining to the discussion on resistive effect in a capacitor

Consider a parallel-plate capacitor with a current $i(t)$ flowing into its positive plate as shown in Fig. 1.3-3. The current entering the positive plate from the left has to deposit charge all along the plate. Therefore the current has to flow through the cross-section of the plate from left to right. The magnitude of current comes down with length traveled towards right. Specifically, the current crossing the cross-section of the plate at mid-point will be about $0.5i(t)$. Thus, there is a linearly varying current crossing the cross-section of metallic electrode at any instant. This current flow meets with the impeding resistance of the metallic plate. Thus there will be a resistive voltage drop along the length of the plate and the plates will no longer be equipotential surfaces. This resistive effect will produce power loss and heating in the capacitor.

There is yet another resistive effect present in a capacitor. A practical capacitor may use some dielectric material (like paper, polyester film, polypropylene film etc) between the electrodes in order to increase the capacitance value. The dielectric substance in between the electrodes will have a non-zero (though very small) conductivity; leading to a leakage current that flows from positive plate to negative plate. Thus the current entering the capacitor gets partially diverted for supplying this leakage current and only the remaining portion is available to create the charge distributions on plates.

A two-terminal capacitor model can model a physical capacitor only if these two resistive effects – one in series and one in parallel – can be neglected. The first resistive component can be made negligible by increasing the thickness of the electrodes and using a high conductivity metal to construct them. These two resistive effects are called the *parasitic resistive effects* in a capacitor.

1.4 Two-terminal Inductance

A moving charge that is part of a steady-current flow (*i.e.,* dc current) in a circuit is in general acted upon by non-electrostatic forces contributed by source regions, electrostatic forces contributed by charge distributions and frictional forces within conductors. A new force called *induced electric force* that acts on such a moving charge makes its appearance in circuits carrying unsteady (*i.e.,* time-varying) currents. This new force component gives rise to a circuit element called *inductance*.

Induced electromotive force and its location in a circuit

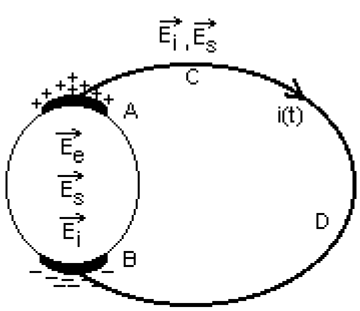

Fig. 1.4-1 A shorted source of electromotive force

Consider a source of electromotive force with a short-circuit across it as in Fig. 1.4-1. The conducting material inside the source is assumed to be of infinite conductivity. The shorting wire is assumed to be made of material of infinite conductivity and the cross-section of the wire is taken to be of near-zero dimensions. These assumptions imply that there is no net force needed to make charged particles move inside the source as well as inside the shorting wire. Further, the static charge distribution on the surface of shorting wire is negligibly small since the wire is very thin.

Therefore, there can be no resistive voltage drops inside the source and the shorting wire. Hence, the current flow in the system will reach infinitely large level if the electromotive force of the source is a steady value - *i.e.,* if the source is a dc voltage source. Of course, the small resistances that are inevitably present within the source and in the shorting wire will limit the current in practice.

Now consider the situation with a time-varying electromotive force in the source. The non-electrostatic field produced by the source in the source region - *i.e.,* $\vec{E}_e$ - is a time-varying quantity. Therefore, the charge distribution on the source terminals will have to be time-varying in order to generate a time-varying electrostatic field inside the source for exact cancellation of time-varying $\vec{E}_e$. This time-varying charge distribution will, in turn, produce time-varying electrostatic field inside the shorting wire, resulting in a time-varying current flow in the wire. However, the conductivity of wire material is assumed to be infinity. Hence, we should expect a time-varying current of infinite magnitude in the wire. But, the current is observed to have finite amplitude in practice. If there is no resistive effect in the wire (conductivity is taken to be infinity), then what is the mechanism responsible for preventing the current from reaching infinitely high value?

The required mechanism arises out of the third component of force of interaction between two charges in arbitrary motion. We had observed in Section 1.1 that this component is dependent on relative acceleration of interacting charges and is given by $\overrightarrow{F_{12ei}} = -\frac{\mu_0 q_1 q_2}{4\pi}\frac{\partial}{\partial t}\left(\frac{\vec{v}_1}{r}\right)\Bigg|_{\vec{v}_2=0}$ N where $\overrightarrow{F_{12ei}}$ is the component of force experienced by charge q_2 due to charge q_1 and $\vec{v}_1$ is the velocity of q_1. We had termed this component of force as the *induced electric force*.

Thus, a moving charge can experience four kinds of force in general – (i) the force due to the non-electrostatic field inside a source acting on it (ii) force due to electrostatic field (iii) magnetic force due to other moving charges (iv) induced electric force from other charges which are accelerating with respect to the location of this charge. The first kind of force will be present only if the charge is inside a source region.

Magnetic force on a moving charge is in a direction perpendicular to the velocity of charge. Hence magnetic force can not change the energy possessed by a charged particle. Therefore magnetic force can not affect the current flow in a circuit though it may produce mechanical forces in current carrying systems. Hence we need to consider only the remaining three forces on a moving charge in circuit analysis.

The net induced electric force experienced by a charge located at a certain point in a circuit carrying *steady current* (- *i.e.,* a dc current) due to all the other moving charges in the circuit will be zero. We accept this statement without proof.

Hence moving charges in a dc circuit do not experience any induced electric force provided there are no other circuits carrying time-varying currents in its vicinity.

However, the net induced electric force experienced by a charge located at a certain point in a circuit carrying *time-varying current* due to all the other moving charges in the circuit *will not be* zero. It will experience an induced electric force that will be proportional to its value. Thus, we can define induced electric field $\vec{E}_i$ at a point as the net induced electric force experienced by +1 coul charge kept at that point. This field exists everywhere in space (including the source region) unlike the non-electrostatic field that is present only within the source.

Now, the force balance on charges inside the source requires that $\vec{E}_e + \vec{E}_s + \vec{E}_i = 0$ since the material inside the source has infinite conductivity. Similarly the force balance condition inside the shorting wire requires that $\vec{E}_s + \vec{E}_i = 0$ since the shorting wire is of infinite conductivity.

Electrostatic field is a conservative field. Hence the work to be done against the electrostatic force in carrying a unit test charge around a closed loop is zero. Induced electric field is non-conservative. Hence the work to be done against the induced electric force in carrying a unit test charge around a closed loop is non-zero.

The electrostatic field inside the source and the shorting wire can be expressed in terms of induced electric field and the non-electrostatic field generated by the source as follows.

Inside the source $\vec{E}_s = -(\vec{E}_e + \vec{E}_i)$

Inside the shorting wire $\vec{E}_s = -\vec{E}_i$

It must be noted that all the field quantities appearing in these equations are functions of space as well as time and that these equations are valid at all points inside the source and wire. Strictly speaking, they should have been expressed as below.

Inside the source $\vec{E}_S(x,y,z,t) = -[\vec{E}_e(x,y,z,t) + \vec{E}_i(x,y,z,t)]$

Inside the shorting wire $\vec{E}_S(x,y,z,t) = -\vec{E}_i(x,y,z,t)$

Let a +1 coul charge be taken around the circuit from B to A through the source and from A to B through the shorting wire. Then,

$\oint -\vec{E}_S(x,y,z,t) \cdot \vec{dl} = 0$ since electrostatic field is conservative. Therefore,

$$\int\limits_{\substack{\text{B to A} \\ \text{through the source}}} -\vec{E}_S(x,y,z,t) \cdot \vec{dl} + \int\limits_{\substack{\text{A to B} \\ \text{through the wire}}} -\vec{E}_S(x,y,z,t) \cdot \vec{dl} = 0$$

But $-\vec{E}_S(x,y,z,t) = [\vec{E}_e(x,y,z,t) + \vec{E}_i(x,y,z,t)]$ inside the source and $-\vec{E}_S(x,y,z,t) = \vec{E}_i(x,y,z,t)$ inside the shorting wire. Therefore,

$$\int\limits_{\substack{\text{B to A} \\ \text{through the source}}} [\vec{E}_e(x,y,z,t) + \vec{E}_i(x,y,z,t)] \cdot \vec{dl} + \int\limits_{\substack{\text{A to B} \\ \text{through the wire}}} \vec{E}_i(x,y,z,t) \cdot \vec{dl} = 0$$

$$\therefore \int\limits_{\text{B to A}} \vec{E}_e(x,y,z,t) \cdot \vec{dl} = -\oint E_i(x,y,z,t) \cdot \vec{dl}$$

Induced electromotive force is present everywhere in the circuit whereas source electromotive force is present only within the source.

The total induced electromotive force acting in a circuit is distributed throughout the circuit.

The quantity on the left-hand side is the electromotive force of the source – that is, it is the work done *by* the non-electrostatic force provided by the source when +1 coul is taken through it from its negative terminal to the positive terminal. Similarly, the quantity on the right-hand side is the *negative* of work done *by* the non-conservative induced electric force when +1 coul is taken through the loop in clockwise direction. That is, it is the *negative* of electromotive force due to the induced electric field in clockwise direction in the loop. We will term this electromotive force as the *induced electromotive force*. Obviously, the current in this circuit suitable magnitude at all instants such that the source electromotive force and induced electromotive force meet each other without leaving any net electromotive force in the circuit loop.

Relation between induced electromotive force and current

The induced electric field at a point in a circuit is the superposition of terms of the form $-\dfrac{\mu_0 q}{4\pi r^2}\dfrac{\partial}{\partial t}\left(\dfrac{\vec{v}}{r}\right)$ where q is the charge per carrier and $\vec{v}$ is the carrier velocity and r is the distance between the carrier and the point. All moving carriers in the circuit are to be considered in the vector summation. Since current in the circuit is related to carrier velocity, we expect the summation to turn out to be proportional to $\dfrac{di(t)}{dt}$ in the circuit. Thus, induced electric field at all points in space will be proportional to $\dfrac{di(t)}{dt}$. Electromotive force of a force field is defined as the *work done by the field* when a unit test charge is taken through a closed path lying in that force field. Hence the induced electromotive force in any closed path will be proportional to $\dfrac{di(t)}{dt}$. The proportionality constant will depend on the spatial geometry of the circuit and the magnetic properties of the medium involved. This proportionality constant is termed as the *inductance* of that closed path. Inductance is designated by the symbol L. If the geometry of the circuit does not vary with time, the value of L will be a constant.

Induced electric field at all points in space will be proportional to $\dfrac{di(t)}{dt}$.

Farady's Law and Induced electromotive force

Faraday's law of electromagnetic induction states that the induced electromotive force in a closed path in a circuit is equal to the time rate of change of flux linkage through that closed path. If the closed path is traversed in counter-clockwise direction and positive flux linkage is defined according to right-hand screw rule, then, this law states that induced electromotive force = $-\frac{d\psi(t)}{dt}$ where $\psi(t)$ is the flux linkage through the closed path in 'Weber-turns' unit.

Faraday's law gives the total induced electromotive force in a closed path. *However, Faraday's law can not tell us where exactly this electromotive force is located.* The discussion in the previous sub-section has shown that the induced electromotive force is distributed all around the closed path.

Determining the polarity of induced electromotive force by using Faraday's law can be confusing at times for a beginner in circuit analysis. Lenz's law is a better option for this purpose. Lenz's law, in effect, states that the induced electric field will be in such a direction that it opposes the change in current that is the cause for appearance of the induced electric field. Refer to Fig. 1.4-1. Assume that the current $i(t)$ is increasing at some time instant. This means that all the charge carriers are accelerating in the direction of current flow at that instant. This acceleration is the cause of induced electric field in the wire and elsewhere. The direction of induced electric field inside the wire will be such that the induced electric force on a positive charge will tend to *deccelerate* it. Thus the induced electromotive force will work against the source electromotive force.

Thus, the induced electromotive force in a closed loop in a circuit with time-varying current is given by $L\frac{di(t)}{dt}$ (for static circuits; see the previous sub-section) as well as by $\frac{d\psi(t)}{dt}$ (by Faraday's law) with the direction of electromotive force as per Lenz's law. Therefore,

$$L\frac{di}{dt} = \frac{d\psi(t)}{dt}$$
$$\frac{d(Li(t))}{dt} = \frac{d\psi(t)}{dt}$$
$$\therefore \psi(t) = Li(t)$$

Inductance of a closed path is the flux linkage in that closed path for unit current.

Thus, inductance of a closed path is the flux linkage in that closed path for unit current. The unit of inductance is Weber-turns/ampere. This unit is given a special name – 'Henry' and is represented by 'H'. Since $L\frac{di(t)}{dt}$ yields an electromotive force, inductance gets another unit – volt-sec per amp. It follows that volt-sec and weber-turns refer to the same physical quantity.

The issue of a unique voltage across a two-terminal element

Refer to Fig. 1.4-2. A time-varying source of electromotive force is connected to a conductor by thin connecting wires of infinite conductivity. The charge distributions at the source terminals and load terminals produce electrostatic field everywhere in space. The electrostatic field inside the source cancels the non-electrostatic field available inside the source and the induced electric field inside the source exactly. (The conductivity inside the source is assumed to be infinity). Electrostatic field inside the connecting wires cancels the induced electric field inside them. Electrostatic field inside the conductor meets the frictional force arising out of collisions of charge carriers with atoms in the lattice and the induced electric force manifesting inside the conductor.

Three issues arise in this context.

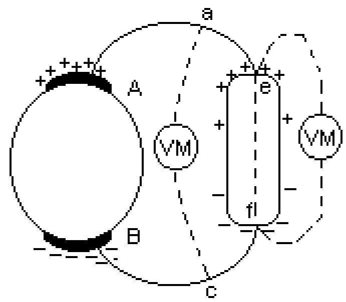

Fig. 1.4-2 Pertaining to uniqueness of terminal voltage of a two-terminal element

(i) The voltage across two points is the electrostatic potential difference between the two points. The voltage across the resistance is given by the potential difference between *e* and *f*. This voltage can be obtained by calculating the work *to be* done in carrying a +1 coul charge from *f* to *e* through the inside of the conductor. But, the electrostatic field inside the conductor is equal to –(frictional force field + induced electric field). Therefore the terminal voltage of resistance will contain a resistive voltage drop plus a term that depends on $\frac{di(t)}{dt}$ (*i.e.*, an inductive voltage drop). Therefore, the conductor can no longer be modeled as a pure two-terminal resistance.

(ii) The voltmeter connected on the right of the conductor attempts to measure the terminal voltage of the resistance right across its terminals. However, the voltmeter connection creates a closed path comprising the resistance element, connecting leads and the meter. This closed path will have induced electric field everywhere inside the connecting leads as well as within the meter. Thus the meter ends up reading the terminal voltage plus the induced electromotive force in the voltmeter leads and in the meter internal circuit. Thus, the reading is in error. The amount of error will keep changing with geometry of voltmeter connection – that is, the reading will be different when the leads are disturbed into a new spatial configuration. The amount of error is dependent on the time-rate of change of flux linkage of the voltmeter loop.

(iii) The voltmeter connected on the left of the conductor reads the actual terminal voltage of the resistance element plus the induced electromotive force in the path *f*-*c*-VM-*a*-*e*. Thus, the reading includes the induced electromotive force in the voltmeter leads and portions of connecting wire in the circuit.

Thus, no unique voltage can be assigned to the resistance by measurement. Therefore, we bring in certain assumptions. The first assumption is that the induced electric field (and, hence, the induced electromotive force too) inside the connecting wires everywhere in the circuit is negligible. The second assumption is that the induced electric field inside the conductor (or inside a capacitor) is negligible. Obviously, this is equivalent to ignoring the inductive effect present everywhere in the circuit.

A unique terminal voltage variable can be assigned for a circuit component only if the induced electric field in the connecting wires and in the space around the components of a circuit is negligible compared to electrostatic field that exists in the space around the components.

A unique terminal current variable can be assigned for a component in a circuit only if the surface charge distributed on the surface of connecting wires is negligibly small at all time.

No circuit can satisfy these assumptions exactly (except in dc circuits). Induced electric field is proportional to rate of change of current. If the rate of change of current is low, then, the strength of induced electric field due to circuit current inside the sources, resistances, capacitors and connecting wires will be low compared to electrostatic field. Thus, the assumptions stated above can be employed if the rate of change of current in the circuit is sufficiently small.

With these assumptions, it becomes possible to model a physical resistor by an ideal two-terminal resistance model and a physical capacitor by an ideal two-terminal capacitance model. Further voltages across a source, resistor and capacitor become unique even with time-varying currents in the circuit.

But, this does not mean that we will not make use of induced electromotive force in a circuit at all!

The Two-terminal Inductance

An electrical device, in general, can have four kinds of force fields that can affect current flow at every point inside the device. They are:

(i) Some non-electrostatic field arising out of some kind of potential energy stored within the device – for instance, the non-electrostatic field generated by chemical potential energy in a dry cell.

(ii) Electrostatic field created by charge distributions on this device as well as other devices nearby.

(iii) Induced electric field created by time-varying currents flowing in the circuit containing this device as well as in neighboring circuits.

(iv) Non-electrostatic force field arising out of frequent collision between moving charged particles and lattice atoms during conduction.

The model used in circuit theory for a device will depend on which of these are strong and which are negligible.

Electrostatic field will be present in all devices in an electrical system and can not be ignored in any device. Electrostatic field inside *any* device is a function of charge distributions on *all* devices in the system. However, if the physical dimensions of the devices are small compared to spatial distance between the devices, then, the electrostatic field inside a particular device is determined uniquely by the charge distribution on its surface alone. Then, there will exist a unique ratio between the electrostatic potential difference across its terminals and the total charge stored on its surface. This is how a two-terminal capacitance can be defined at all.

Thus, a two-terminal capacitance is a model for an electrical device that has only electrostatic field inside in it and the electrostatic field inside depends only on its own charge distribution. The non-electrostatic field existing in the metallic electrodes when current flows in them is ignored in an ideal two-terminal capacitance. The induced electric field that exists inside the device due time-varying currents everywhere is also ignored in an ideal two-terminal capacitance.

An ideal two-terminal capacitance has no induced electric field inside.

A piece of conductor with finite conductivity carrying a current will have electrostatic field, non-electrostatic field arising out of frictional forces and induced electric field due to time-varying currents in the circuit as well as in other circuits. An ideal two-terminal resistance models this piece of conductor by ignoring (i) the current component that is needed to build a time-varying charge distribution on its surface and (ii) the induced electric field inside the conductor.

An ideal two-terminal resistance has no induced electric field inside.

Circuit Theory models a piece of connecting wire by ignoring all fields that exist within the wire and taking all of them to be zero at all instants. Thus, Circuit Theory assumes that there is no resistive drop across connecting wire; there is no induced electromotive force in connecting wire and there are no charges distributed on the connecting wire. Such an element is called the *ideal short-circuit element*.

An ideal connecting wire (or short-circuit element) has no field of any kind inside.

An electrical source will have all the four kinds of fields inside. However, the *ideal two-terminal source model* of Circuit Theory attempts to model such a source by (i) ignoring the non-electrostatic field arising out of friction within conductor (ii) ignoring the induced electric field inside in comparison with electrostatic field and (iii) ignoring the component of current needed to build a time-varying charge distribution at its terminals.

An ideal two-terminal source has no induced electric field and no non-electrostatic field arising out of friction against carrier movement.

And, the *ideal two-terminal inductance model* of Circuit Theory is a model for an electrical device in which there are only two fields – the induced electric field and the electrostatic field. It is not a source and hence there is no source field. It uses conducting substance and hence there is a non-electrostatic field arising out of collisions of charge carriers with lattice atoms when a current flows through it. But this field is ignored in comparison with the other fields. Further, the component of current needed to build a time-varying charge distribution on its surface is assumed to be negligibly small.

An ideal two-terminal inductance has no non-electrostatic field arising out of friction against carrier movement.

Consider a long piece of round conductor carrying a time-varying current as shown in (a) of Fig. 1.4-3. This wire is not a connection wire. It has a non-zero cross-sectional area. But it is indicated by a line in Fig. 1.4-3. The current entering the conductor is $i(t)$ and the same current leaves the conductor at far end. The value of current crossing any cross-section at a particular instant will be the same everywhere since we neglect retardation effect as well as the current that is required to build the surface charge distribution.

There is induced electric field at all points within this conductor. The induced electric field at a point inside is the sum of terms of the form $-\frac{\mu_0 q}{4\pi}\frac{\partial}{\partial t}\left(\frac{\overrightarrow{v(t)}}{r}\right)$ where q is the charge per carrier and $\vec{v}(t)$ is the carrier velocity and r is the distance between the carrier and the point – as many terms as there are moving carriers in the conductor. All the charge carriers will be moving with same instantaneous velocity that is proportional to $i(t)$. But the distance between the point at which the induced electric field is calculated and the location of carrier (*i.e.*, r) will be large for all those carriers that are moving at a far away location at the instant under consideration. Therefore, only those carriers that are presently moving within the immediate vicinity of the point at which field is being calculated will contribute to the induced electric field significantly. Thus the induced electric field will be relatively low everywhere, and, correspondingly the total induced electromotive force in the long conductor will be relatively low. The induced field as well as the total induced electromotive force will be proportional to $\frac{di(t)}{dt}$ since $\frac{\partial}{\partial t}\left(\frac{\overrightarrow{v(t)}}{r}\right)$ that appears in the equation for induced electric field due to a moving charge is directly related to $\frac{di(t)}{dt}$.

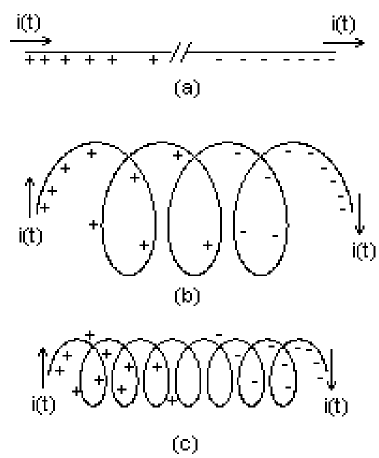

Fig. 1.4-3 Towards a two-terminal inductance

The conductor is assumed to be of large conductivity. Then, the net force experienced by a charge carrier inside must be zero. Therefore, the induced electric field at every point within the conductor will be cancelled exactly by the electrostatic field created by the surface charge distribution. This charge distribution is shown in Fig. 1.4-3 assuming that $\frac{di(t)}{dt}$ is positive at the instant under consideration.

A physical inductor is constructed so as to strengthen the induced electric field and the induced electromotive force inside the conductor forming the inductor.

Refer to (b) of Fig. 1.4-3. The same conductor is wound into a coil of 4 turns. Now the relative distances between moving carriers in various sections of the wire are reduced considerably. Hence the induced electric field at any point in the conductor will have a value greater than the value when the entire conductor was stretched out in a straight-line as in (a). Therefore, the total induced electromotive force will also be higher. Obviously, the value of induced electromotive force will go up further if the turns can be kept closer.

Refer to (c) of Fig. 1.4-3. The same conductor is wound into a coil of lower diameter and higher turns. And the turns are kept closer. This structure will have still higher induced electric field everywhere. The total induced electromotive force will also be higher. If the wire has an insulation cover the turns can touch each other.

Thus, winding a long length of wire into an optimally sized and layered coil with turns touching each other will result in large induced electric field everywhere in the wire and large induced electromotive force over the length of the wire when the current through the coil is time-varying. The induced electric field everywhere inside will be cancelled by the electrostatic field created by the surface charge distribution all along the wire surface.(The conductivity of wire material is assumed to be very large). Therefore, the electrostatic potential difference between the ends of the coil - *i.e.,* the voltage difference between coil terminals – will be equal to the total induced electromotive force in the coil. The polarity of voltage will follow Lenz's law.

What we have described here is an air-cored coil. Air-cored inductor is essentially a long piece of wire that is arranged to occupy a small region of space of dimensions that are very small compared to its length. Such a spatial confinement of a long wire results in strengthening of induced electromotive force in it. Further strengthening of induced electric field inside the wire can be attained by winding it around a core made of magnetic material (usually iron). If the core made of magnetic material is a closed structure, the induced electric field will be enhanced further. Moreover, a closed core structure confines the time-varying magnetic field to the core itself and reduces the magnetic flux linking rest of the circuit to negligible levels.

A physical inductor that is designed to strengthen the induced electric field within itself, while confining the time-varying flux-linkage to predominantly within itself, can be modeled by an ideal *two-terminal inductance model* provided the resistive voltage drop in the coil can be neglected and the capacitive effect due to surface charge distribution over the coil surface can be neglected.

The value of inductance depends on the geometry of the coil and core assembly and the magnetic properties of the core. *Inductance of a coil is proportional to the square of number of turns of the coil, area of a turn and magnetic permeability of the core material.*

The symbol and variable assignment for an ideal two-terminal inductance is shown in Fig. 1.4-4.

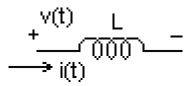

Fig. 1.4-4 A two-terminal inductance

The governing equations of a linear two-terminal inductance are:

$$\psi(t) = Li(t)$$
$$v(t) = L\frac{di(t)}{dt} \qquad (1.4\text{-}1)$$
$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)dt = \frac{1}{L}\int_{-\infty}^{0} v(t)dt + \frac{1}{L}\int_{0}^{t} v(t)dt = i(0) + \frac{1}{L}\int_{0}^{t} v(t)dt$$

Voltage-Current relation of a two-terminal inductance

where $\psi(t)$ is the flux-linkage at t in Weber-turns, $v(t)$ is the voltage across the inductance and $i(t)$ is the current entering the higher potential terminal. $i(0)$ is the current in the inductor at $t = 0$.

A coil can have induced electric field and induced electromotive force present in it due to accelerated motion of charges (*i.e.*, time-varying current) in the circuit in which it is connected and/or due to accelerated motion of charges taking place in another physically separated circuit. The electromotive force induced in the coil due to its own time-varying current is termed as *self-induced electromotive force* and the electromotive force induced in it due to current in another circuit is termed as *mutually induced electromotive force*. Self-induced electromotive force is associated with an inductance value called *self-inductance*. Eqn. 1.4-1 describes the governing equations of self-inductance.

Self-induction vs. Mutual Induction

There is no region without induced electric field and induced electromotive force in any circuit carrying time-varying currents. All devices and components of such a circuit are affected by electromagnetic induction. Thus, all devices have a little inductive effect associated with them. The associated inductance will be called the *parasitic inductance* of the two-terminal element (unless it is a two-terminal inductance). Ideal two-terminal element models ignore the parasitic inductance in a resistor, capacitor, source and connecting wire.

1.5 Ideal Independent Two-terminal Electrical Sources

Electrical sources are devices that are capable of applying a non-electrostatic force on a charge that moves through the source region. They can deliver energy to the charged particle or absorb energy from it.

Ideal Independent Voltage Source

A two-terminal voltage source will have a non-electrostatic field at every point inside the source region. The charge distribution on the terminal surfaces of the source will create an electrostatic field at all points inside the source. The two fields cancel each other at all points at all instants under all conditions if the material inside the source is of infinite conductivity. The terminal voltage (which is an electrostatic potential difference) will always be equal to the internal electromotive force in that case.

The conducting material inside the source will have finite conductivity in practice. Charge carriers moving inside such material require net non-zero force to work against collisions with lattice atoms. This will call for a difference between the internal non-electrostatic field and the electrostatic field to exist. Then, the terminal voltage will